ANNALS OF THE UNIVERSITY OF ORADEA
FASCICLE OF TEXTILES, LEATHERWORK

THE FOURIER SERIES USED IN ANALYSE OF THE CAM MECHANISMS FOR THE SHOEMAKING MACHINES (PART I)

IOVAN-DRAGOMIR Alina[1], DRIȘCU Mariana[2]

[1, 2] „Gheorghe Asachi" Technical University, Textile, Leather and Industrial Management Faculty, Str.Prof.Dr.Doc. Dimitrie Mangeron, nr.28, 70050, Iasi, Romania, E-Mail: adragomir@tex.tuiasi.ro

Corresponding author: Iovan-Dragomir, Alina, E-mail: adragomir@tex.tuiasi.ro

Abstract: *A computer assisted procedure for the cinematic analysis of the mechanism of a cam is essential in making a certain type of research operations. They mainly refer to the optimization of operations running on specific machinery, or to the re-design of the mechanism, in order to make the mechanism digital. This analysis seems even more important, when we consider the fact that most of the machines used in shoe industry nowadays use a cam mechanism.*
The paper is devided in two parts.
In first part, it is elaborated a method of finding of a function G(x), belonging to a Fourier series, which approximates the numerical values $\{x_i, y_i\}$*, with the biggest accuracy. Finding the function that approximates the most accurately the data set, for the position parameters of the follower* $S(\omega)$*,* $\Psi(\varphi)$ *will lead to a complete kinematic and dynamic analysis of the cam mechanism. These values repeat with* $T = 2\pi$ *period.*
In second part, the method is tasted using MatCAD work sessions which allow a numerical and graphical analysis of the mathematical relations involved, in order to test the reability of the method. The set of experimental data are resulted after measuring a cam mechanism of a machine used in shoemaking.

Key words: *Fourier series, cam, machine, shoemaking,*

1. INTRODUCTION

Cam mechanisms are involved in construction of many machines for shemaking production, as are: lasting machines, steaching machines and so one. The analysis of a cam mechanism means determining these position parameters of the follower, as well as determing its momevent law.

Taking into account the fact that the follower always makes an alternative-period move, we will mathematically model the position parameters of the follower (its movement S and the rotation angle, Ψ) by using Fourier series (fig. 1).

To this aim, the paper presents theoretical analysis of: Fourier series, numerical calculus methods for a definite integral necessary for the determination of the Fourier coeficients, establishing optimization criteria in order to obtain the function that best approximates the position parameters of the follower.

2. GENERAL INFORMATION

2.1 Cam mechanisms

Cam mechanisms [1] are divided in two groups: planar and spatial. In case of the planar mechanisms, the cam and the follower roll also, have the movements in the same plan or in parallel plans. If the movements of the elements of the mechanism are in different plans, this is a spatial mechanism.

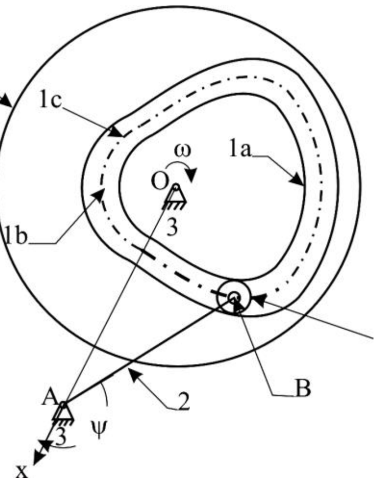

***Fig. 1**: Cam mechanism*

The mechanism presented in fig.1 is composed of: plan cam with groove 1 (bilateral contacts), oscillating follower roll 2 and the framework 3. The contact between the follower and the cam is made by a roll, which go in the groove of the cam.

The cinematic analysis of a cam mechanism can be made using the follow methods [5]: the method of the cinematic charts, the method of the vectorial equations and the method of the transformation of the cam mechanism in the mechanism with lower pairs.

2.2 Fourier series and functions

From the special literature one knows that a periodical function **f(x):[0,T]** with values in *R* can be approximated with **Fourier** [2], [3] series, according to the relation:

$$F(x) := \frac{a_0}{2} + \sum_n \left[a_n \cdot \cos\left[\frac{(2\,n\cdot\pi\cdot x)}{T}\right] + b_n \cdot \sin\left[\frac{(2\,n\cdot\pi\cdot x)}{T}\right]\right] \quad \textbf{(1)}$$

From this relation one can notice that, the Fourier series are an infinite sum of trigonometric functions sin, cos with arguments increasing in arithmetical progression and multiplied by a_n and b_n. To these one adds the a_0 term divided by 2.

The forms of the coeficiets of the the Fourier, [2], [3], [4], [6], are:

$$a_0 := \left(\frac{1}{T}\right) \cdot \int_0^T f(x)\,dx \quad \textbf{(2)}$$

$$a_n := \frac{2}{T} \cdot \int_0^T f(x) \cdot \cos\left[\frac{(2 \cdot n \cdot \pi \cdot x)}{T}\right] dx \quad \textbf{(3)}$$

$$b_n := \frac{2}{T} \cdot \int_0^T f(x) \cdot \sin\left[\frac{(2 \cdot n \cdot \pi \cdot x)}{T}\right] dx \quad \textbf{(4)}$$

Where n is a number having values from 0 and ∞.

For the period is T=2*π, the a_0, a_n and b_n are the Fourier series coefficients and, according with special literature, they have the form:

$$a_0 := \left(\frac{2}{\pi}\right) \cdot \int_0^{2 \cdot \pi} f(x)\, dx \quad \textbf{(5)}$$

$$b_n := \frac{1}{\pi} \cdot \int_0^{2\pi} f(x) \cdot \sin(n \cdot x)\, dx \quad \textbf{(6)}$$

$$b_n := \frac{1}{\pi} \cdot \int_0^{2\pi} f(x) \cdot \sin(n \cdot x)\, dx \quad \textbf{(7)}$$

and Fourier series will be:

$$F(x) := \frac{a_0}{2} + \sum_n \left(a_n \cdot \cos(n \cdot x) + b_n \cdot \sin(n \cdot x)\right) \quad \textbf{(8)}$$

This is an infinite sum of trigonometrical functions sin, cos multiplied by coefficients a_n and b_n calculated for different values of the argument *x*.

For a finite terms number of the series given by relation (8), Fourier functions will obtain. Hence, a **Fourier function** will have the following form:

$$G(x) := \frac{a_0}{2} + \left[\sum_{n=1}^{nf} \left(a_n \cdot \cos(n \cdot x) + b_n \cdot \sin(n \cdot x)\right)\right] \quad \textbf{(9)}$$

where:

- nf represents the number of terms of the series;
- n is a variable having values from 1 to *nf;*

- a_0, a_n and b_n are the Fourier series coefficients calculated with the relations (5), (6), (7) for n in the range [0, nf].

NOTE

The number nf, defining the Fourier function form, is choosing as function of convergence criterion of the series, so that the function G(x) approximates the function F(x) with the best accuracy.

3. RESULTATES

3.1 Method of approximation for a periodical experimental data set

In the most situations, a data set $\{y_i, x_i\}$ can not be approximated with an elementary analytical function f(x). For this reason, in computer assisted design, one applies to modern mathematical methods providing methods of finding of the functions which drive to the approximation of the data set.

In this paper, one presents a method of finding of the function G(x), belonging to a **Fourier series**, which approximates the numerical values $\{x_i, y_i\}$ with the biggest accuracy. These values repeat with $T = 2\pi$ period.

The work routines of the present methodology are:
- calculation of the Fourier series coefficients;
- execution of the integral calculus using the trapezes method;
- determining the Fourier function.

Calculation of the Fourier series coefficients

From the relation (2), (3), (4) or (5), (6), (7) an one observes that, for the coefficients a_0, a_n and b_n obtaining, one use an integral calculus from an analytical function **f(x)** or **f(x)cos(nx)** and **f(x)sin(nx)**. As long as the values of these functions are unknown, the integral calculus will apply to numerical methods of determination of a finite integral. According to these, the numerical value of the finite integral represents the area of the field limited by axis *Ox*, function g(x) and the segments of bee line: x=a and x=b, having the following relation:

$$\int_a^b g(x)\,dx := \sum_i g\left(x_i\right)\cdot h_i \qquad \textbf{(10)}$$

where:
- x_i – values of the independent variable for the range [a, b];
- $g(x_i)$ - values of the function *g(x)* in the points x_i ;
- h_i – the distance between two consecutive values x_i, x_{i+1} like: $h_i=x_{i+1}-x_i$

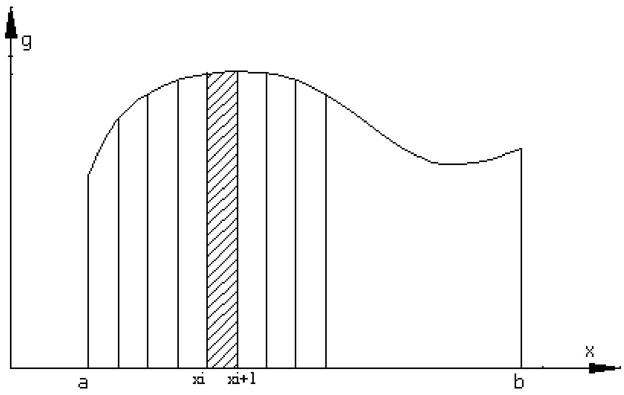

Fig.2: *The area of the field limited by axis Ox, function g(x) and the segments of bee line: x=a and x=b*

Execution of the integral calculus using the trapezes method

In the case of the periodical data set with period 2π, whose values are obtaining from 10° to10°, for example, the numerical data for the integral calculation will be:

$a=0 \quad b=2\pi \quad h_i=2\pi/360 \quad g(x_i)=Y_i$.

The professionals consider that the trapezes formula is the most accurate one for the integral calculation, that can be found in the special literature. Based on this methodology, for a data set Y_i determined in the points x_i= 0, 10°, 20°, 30°..360°, the relation (7), can be re- written as:

$$S := \left(\frac{h}{2}\right)\cdot\left(Y_0 + Y_{36} + \sum_{i=1}^{35} Y_i\right) \tag{11}$$

From the above presentation, the relations for the coefficients a_0, a_n and b_n determination, the relations used are:

$$a_0 := \left(\frac{1}{36}\right)\cdot\left(Y_0 + Y_{36} + \sum_{i=1}^{35} 2\cdot Y_i\right) \tag{12}$$

$$a_n := \left(\frac{1}{72}\right)\cdot\left(\sum_{i=1}^{35} 2\cdot Y_i\cdot\cos\left(\phi r_i\right) + Y_0 + Y_{36}\right) \tag{13}$$

$$b_n := \left(\frac{1}{72}\right) \cdot \sum_{i=0}^{36} 2\, Y_i \cdot \sin\left(\phi r_i \cdot n\right) \tag{14}$$

where:

- Y_i are experimental numerical values;
- Φr_i – angle belonging to the range $[0,2\pi]$, rad.

Determining the Fourier function

In this routine, one determines the number of terms of the series which will approximate with the biggest accuracy the routine of points $\{x_i, y_i\}$ obtained in the sessions of the getting of data, which will determine Fourier function. To this aim, one calculates with the relation:

$$F_i := \frac{a_0}{2} + \left[\sum_{n=1}^{nf} \left(a_n \cdot \cos\left(n \cdot \phi r_i\right) + b_n \cdot \sin\left(n \cdot \phi r_i\right)\right)\right] \tag{15}$$

the values of the **Fourier function** for a finite variable number *nf* of terms and one selects that *nf* for which the difference between the initial values Y_i and F_i are minimum.
It results that the selection criterion of the number of terms of the series, and therefore of the Fourier function is:

$$| Y_i - F_i| < \varepsilon \tag{16}$$

where ε is dependent of the application nature.

The elaborate method will be use in MathCAD sessions, in order to analyse cam mechanisms. The set of experimental data are resulted after measuring a cam mechanism of a machine used in shoemaking.

REFERENCES

[1] D.B.Marghitu - editor, *Mechanical Engineer's Handbook*, ACADEMIC PRESS, USA, 2001.

[2] H. Dym, H.P. Mckean, *Fourier Series and Integrals*, Academic Pr., 1986.

[3] James Ward Brown, Rull V., *Fourier Series*, Churcill, McGraw_hill, 2001.

[4] Boris Osilenker, *Fourier series in orthogonal polynomials*, Mathematical Reviews, 2001.

[5] A. Iovan-Dragomir, *Elemente de proiectare a mașinilor din industria confecțiilor din piele*, Editura PERFORMANTICA, 2013.

[6] Cocea, M., and Buzescu L. F., *Conferinţă internaţională de comunicări ştiinţifice, Tehnologii Moderne, calitate, restructurare*, Chişinău- Iaşi, 2005.

[7], D.F. Croitoru, A. Dragomir, *Utilaje pentru industria confecţiilor din piele,* Iasi, 2003.